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ABSTRACT

This paper provides an historical account of the development of the International System of Units (SI), a complete listing of these units, and rules concerning their use and proper abbreviation. Ambiguities concerning the use of the system are explained. Appendices contain conversion factors for U.S. - British to SI units along with several mechanics problems showing the correct use of SI units. (CP)

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USING SI UNITS IN MECHANICS

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SE 020 319

I. Introduction

Engineering teachers in the United States cannot afford to postpone much longer the use of SI metric units in their classes. In fact it appears to many of us that we are already a year or two behind in introducing to our students the system of units which they will be using for most of their professional lives. Four years hence, beginning with their initial employment following graduation, most all of our current first-year students who engage in technical engineering work will be dealing with quantities expressed in the SI system of units.

Major segments of US industry have already made or are in the process of making the transition, and within the next two or three years we should see the trend "snow ball." Engineering teachers have the responsibility of preparing their students for the environment in which they will find themselves, and this environment is rapidly becoming a metric one.

II. History

In 1875 the Convention of the Metre, in which the United States and sixteen other nations participated, established by treaty (a) a permanent International Bureau of Weights and Measures (IBPM) located near Paris, France, (b) an international General Conference on Weights and Measures (CGPM), and (c) an international Committee on Weights and Measures (CIPM). The CIPM implements recommendations of the CGPM and directs the work of the IBPM.

At the 11th meeting of the CGPM in 1960 the International System of Units (abbreviated SI in all languages and standing for the French "Le Système International d'Unités") was officially adopted. Subsequent meetings have resulted in a number of redefinitions, additions of

new units, and other changes. The CGPM recommends the world-wide use of the SI system for all technical, scientific, practical, and educational purposes.

The United States and several small countries of no industrial consequence are the only nations in the world which have not officially adopted the SI system as a national policy. The governments of most English-speaking countries which have adopted the SI system have taken the initiative to organize and expedite an orderly transition. Such has been the case, for example, in Great Britain, Australia, New Zealand, and Canada. The United States is conspicuous in the eyes of the world for not officially adopting the SI system and for the lack of governmental leadership in organizing the change-over. Congress has not been sufficiently aware of the national importance of adopting the SI system and has, consequently, failed to pass enabling legislation. There may also be a question as to whether the recommendation made to Congress by the National Bureau of Standards was sufficiently forceful. Despite the lack of government leadership the United States is well on its way toward the change-over to SI units primarily as a result of the initiative taken by major segments of private industry which, more and more, must interface with the metric practice of foreign markets.

III. Introduction of SI Units

It is quite apparent that our engineering schools should undertake a transition to SI units without further delay. Unless we move ahead soon, our students will have every justification to criticize their schools for not preparing them in this important regard. There are two educational areas which must be treated differently during the transition period. The first area concerns the basic engineering-science

courses where SI units can and should now be pursued. The second area involves our design courses where a full treatment in SI units depends on the availability of standard design information such as structural shapes, fasteners, gearing, etc. Among the engineering science courses nowhere does the adoption of the SI system have a greater impact than in mechanics, upon which so much of the analytical structure of engineering depends.

One of the first questions to arise is whether to introduce SI units on, say, a 50-50 basis with US-Customary units or whether to use SI units predominantly or exclusively. Although we will undoubtedly be testing each approach in the months ahead, we should be aware of the conclusions of the Australian and New Zealand engineering teachers who are virtually unanimous in their advice to teach mechanics with essentially 100 per cent SI units. During the initial phases of their transition period those who tried a dual-unit approach reported that the extra bother and confusion introduced by the dual units detracted from the focus on basic principles and formulation techniques. Once the basic ideas of mechanics were mastered, they report, the student can easily make a conversion to another set of units. There are several schools in this country which have already made their decision to present mechanics and other subjects using predominantly SI units, and we should observe their experiences.

IV. Summary of SI Units for mechanics

Contrary to frequent belief the SI system is neither the CGS system nor the MKS gravitational system. In the latter system the kilogram is used both as a unit of force and a unit of mass. This dual use is not permitted in the SI system. The units in the SI system are divided into three categories:

Base units	Name	SI-symbol
length	metre (er)	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
luminous intensity	candela	cd
amount of substance	mole	mol
Supplementary units		
plane angle	radian	rad
solid angle	steradian	sr
Derived units with special names (selected)		
force	newton	N (= kg·m/s^2)
pressure, stress	pascal	Pa (= N/m^2)
work;energy	joule	J (= N·m)
power	watt	W (= J/s)
frequency	hertz	Hz (= 1/s)
potential difference	volt	V (= W/A)
resistance	ohm	Ω (= V/A)

Derived quantities without special names (selected)

(Quantity)	(SI units)
moment of force	N·m (= kg·m^2/s^2)
acceleration (linear)	m/s^2
acceleration (angular)	rad/s^2
velocity (linear)	m/s
velocity (angular)	rad/s
moment of inertia (area)	m^4 or (mm^4)

moment of inertia (mass)	$kg.m^2$
impulse (linear)	N·s (= kg·m/s)
impulse (angular)	N·m·s (= $kg.m^2/s$)
momentum (linear)	kg·m/s (= N·s)
momentum (angular)	$kg.m^2/s$ (= N·m·s)
density	kg/m^3 or t/m^3
specific weight	N/m^3 (= $kg/(s^2.m^2)$
viscosity (absolute)	$N.s/m^2$ (= Pa·s)

Additional Acceptable Units

Quantity	Name	SI symbol
time	day	d
	hour	h
	minute	min
mass	metric ton	t (= 1000 kg)
angle	degrees (decimals)	°
distance (nav.)	nautical mile	- (= 1.852 km)
speed	knot	- (= 1.852 km/h)

V. Characteristics of SI-system of units

1. The SI-system is a coherent system of units, i.e. a unit quantity results upon the combination by multiplication or division of two other unit quantities. For example in F = ma, (1 N) = (1 kg)(1 m/s^2). Thus no constants are required to account for the adjustment of numerical values needed in a noncoherent system.
2. The SI-system is an absolute system with mass as a base unit in contrast with the gravitational system where force is a basic unit and mass is derived.

3. Only one unit exists for each physical quantity. This unique use for each unit will help to avoid confusion which arises when one named unit, such as pound, is used for force and mass. In the SI system the kilogram (kg) is used exclusively for mass and never for force, contrary to popular usage.
4. There is only one accepted symbol (or abbreviation) for each physical quantity. For example "s" and not "sec" denotes time in seconds and "W" and not "w" stands for power in watts.
5. The convenience of the decimal system is inherent in the metric system, and the elimination of fractional designations will greatly simplify the communication of measurements.

VI. Selected rules for the consistent use of SI units

1. Spacing. A space is always left between a numerical value and its unit symbol whereas no space is used between an algebraic quantity and its numerical coefficient. Also when printed the symbol for a unit is set in vertical letters, whereas the algebraic quantity is in italics. Thus

   12 m means a distance of 12 metres and

   12*m* means 12 times the algebraic quantity *m*.

2. Capitalization. When units are written out in words, they always start with lower case letters, except at the beginning of a sentence. But when they are expressed as a symbol, they are capitalized when the name of the unit is derived from the name of an individual. Thus we write

   10 newtons or 10 N

   40 watts or 40 W

   6 metres or 6 m

3. Plurals of units. When units are written out in words, the plural is used in the usual grammatical sense, but plurals are never used with unit symbols. Thus

   4 kilograms or 4 kg never 4 kgs

4. Periods. Except to mark the end of a sentence a period is never used after a symbol of an SI unit.

5. Decimal point. Decimal points are placed on the line, as currently practiced in the United States. The frequent European practice of using a comma in place of the decimal point or in placing the decimal point at mid-line height is discontinued.

6. Number grouping. Commas will not be used to separate numbers representing multiples of 1000. Instead a space is to be used on both sides of the decimal point. Thus a correctly written number is

   42 076.326 47

   Except where consistency with other numbers in a table of numerical values is called for, 4-digit numbers need not include a space. Thus we may write

   2679 in place of 2 679

   0.4186 in place of 0.418 6

7. Prefixes. It is recommended that, in general, numerical values be kept between 0.1 and 1000 by the use of the appropriate prefix multiplier. As a general rule, prefixes which shift the decimal point less than three places, hecto (h, 10^2), decka (da, 10), deci (d, 10^{-1}), centi (c, 10^{-2}), are to be avoided except for some measurements of areas and volumes.

The most common prefixes are listed here:

Factor	Prefix	SI Symbol
$1\ 000\ 000\ 000 = 10^9$	giga	G
$1\ 000\ 000 = 10^6$	mega	M
$1\ 000 = 10^3$	kilo	k
$0.001 = 10^{-3}$	milli	m
$0.000\ 001 = 10^{-6}$	micro	μ
$0.000\ 000\ 001 = 10^{-9}$	nano	n

Examples $4250\ N = 4.25(10^3)\ N = 4.25\ kN$

$0.026\ m = 26(10^{-3})\ m = 26\ mm$

$0.017\ \mu s = 17(10^{-3})\ \mu s = 17\ ns$

Double prefixes are to be avoided. Thus use

GN not kMN

ns not μms

8. Combinations of units. Avoid using prefixes in the denominator of compound units. The one exception is kg which is a base unit. Thus write

kN/m and not N/mm

kg/s and not g/ms

J/kg and not mJ/g

The ambiguous double solidus is to be avoided. Thus write

m/s^2 or $m \cdot s^{-2}$ and not m/s/s

$J/(s \cdot m^2)$ or $J \cdot s^{-1} \cdot m^{-2}$ and not $J/s/m^2$

When units appear to a power, such as square millimeters, the exponent refers to the entire unit. Thus mm^2 means $(mm)^2$ and not milli square metre $m(m^2)$.

9. Calculations in base units. In general it is recommended that calculations be carried out in base units and answers be converted

to convenient size numbers by the use of appropriate prefixes. It is worth noting, however, that certain simplifications will be frequently utilized. For example

moment (N·m) = force (kN) x distance (mm)

force (kN) = mass (kg) x acceleration (km/s^2)

force (kN) = mass (g) x acceleration (m/s^2)

VII. Difficulties. The introduction of the SI system poses certain difficulties which should be recognized so that they may be dealt with adequately.

1. Absolute vs. gravitational. The engineer has found it convenient to use a gravitational system of units by virtue of the frequency with which he deals with the direct measurement of force. Generations of engineering students have been educated with the acceptance of the gravitational basis for measurement. Thus the fundamental shift to mass as the basic unit with force as a derived unit is a significant change in point of view which will require time and adjustment, and which, in effect, is a forced concession by the engineer.

2. Weight and Mass. In the gravitational system the kilogram has been used both as a unit of force and as a unit of mass, just as the pound finds dual use in US-Customary units as lbf or lbm. Now that the kilogram under the SI system is to be used exclusively as a unit of mass, frequent misuse of kilogram for force is bound to occur for some time.

The most troublesome area of misuse lies in the distinction between "weight" and "mass," which has occasioned misunderstanding for many years. Most teachers of mechanics have consistently

defined "weight" as the force applied to a mass due to gravitational attraction. It is doubtful if this commonly accepted technical meaning will change, and therefore, the weight of an object should henceforth be expressed in newtons. However, when using the gravitational system we frequently speak of the weight of an object when we really mean mass. In the vast majority of cases we should now specify the mass of a body rather than its weight. In the analysis of static structures specification of mass necessitates an extra step to calculate the weight by multiplying the mass by g (9.81 m/s^2). In the analysis of accelerating bodies the specification of mass is, of course, more direct and avoids the necessity of a separate calculation of mass.

The CGPM has recognized the confusion between "weight" and "mass" and has recommended the gradual disappearance of the word "weight" to be replaced by "mass" where mass is intended. Whereas the author agrees that we should now use mass where mass is intended, he does not believe that the word "weight" should or will disappear from our language. The most common force in existence is that of gravitational attraction, and there has been a word or an expression for this force in most, if not all, languages for countless centuries. It is very doubtful if the recommendation of CGPM will change this expression. Also, to substitute a phrase such as "the force of gravitational attraction on" for "weight" every time we write about or speak of this force is entirely too awkward.

The accepted use of words cannot be overlooked. The word weight not only expresses gravitational attraction, but it also refers to a physical object. We speak of a counter-weight meaning

an object attached to a machine element or a structure for balancing purposes. We speak of a paperweight or the placement of a weight on the table. This use of weight to refer to an object as well as the property of gravitational force is unavoidable.

To the layman the distinction between weight and mass is of little or no consequence. Our grocery-store scales will soon be recalibrated to indicate the number of kilograms of mass being purchased instead of pounds of mass. The butcher, however, will continue to "weigh" the hamburger. When one "weighs" himself, he is actually interested in determining his mass. One of our manufacturers of scales should rephrase their slogan to read "honest mass, no springs."

3. Double meaning. The symbol m has the dual meaning of milli and metre wh[illegible] unfortunate but now unavoidable. This could lead to ar[illegible] the multiplication of certain units. For example [illegible]mit the writing of products such as moment either as [illegible] where a space replaces the dot. Thus if the order of the multiplication is reversed, the distinction between m N (metres times newtons) and mN (millinewtons) is virtually lost. Similarly m·s (metres times seconds) can easily become ms (milliseconds) without use of the dot. Thus, if the dot is used consistently, we can avoid this possible ambiguity.

As was indicated in Section VI-1, an ambiguity can arise when an entry of linear measurement such as 12 m could be mistakened for 12m meaning twelve times a mass m.

These several examples of notational difficulties emphasize the need for careful and consistent observance of the rules

governing the use of SI symbols.

It may be observed that units as written in US-Customary units are two or more letter abbreviations which are not easily confused with single symbols for algebraic quantities. Thus 4.2 ft is unmistakable whereas 4.2 m might easily be confused with 4.2m which has an entirely different meaning.

4. Size of units. One of the disadvantages of the SI system is the awkward size of certain of the units. Thus 1 N is only slightly more than 0.2 lb and is too small for many specifications. Consequently the kilonewton (1 kN = 224.8 lb) will find frequent use.

   Probably the most awkward sized unit is the pascal for pressure or stress, since it takes 6895 pascals to equal one pound per square inch. Fluid pressures will be frequently expressed in kPa (100 lb/in^2 = 689.5 kPa, and mechanical stress will be expressed in MPa (10,000 lb/in^2 = 68.95 MPa). The elastic modulus of steel is 29.0(10^6) lb/in^2 = 200 GPa.

5. Misuse. There is bound to be misuse of the SI system in uninformed quarters. Many European engineers, who have used the MKS gravitational system for years will find it difficult to restrict their use of the kilogram to mass. Current industrial specifications for various pieces of equipment in Europe are frequently expressing the "weight" of the product in kilograms. One well-known U.S. manufacturer of precision testing machines has published a slide rule for quick conversion from lbf to kgf and from psi to kg/m^2, thereby violating the restricted and precise SI meaning of the kilogram.

C. Proposed changes. Two changes proposed unofficially by National Bureau of Standards personnel are aimed at clarification but, in the opinion of the author, should be rejected. The first proposal is to specify the units of moment or torque as joules per radian (J/rad) in an attempt to avoid confusion with the base units for work and energy, N·m. The rationale is that work is moment times angle, so that moment is work divided by angle. The author rejects this proposal on two grounds. First, the existence of moment does not require angular displacement, and, second the joule (J) should be used for work and energy which will distinguish it from N·m when used for moment.

The second proposal is a radical departure from the well-established meaning of weight as the absolute force exerted on a body by gravitational attraction. The proposal is to define "weight" as that force required to support a body at rest relative to the coordinate system in which the force is measured. Thus in an elevator descending with an acceleration g a "weightless" condition would be observed. This proposed redefinition of weight is contrived to accommodate the apparent loss (or gain) in "weight" encountered in accelerating systems. This definition would disassociate "weight" from gravitational attraction and would do a disservice to the engineer. Rejection of this proposal is urged.

VIII. Conversion. One can not learn the SI system by depending on converting from US-Customary units to the corresponding metric values. It is essential that we learn to think directly in SI terms, which require that we develop a feel for the size and significance of SI units. During the period of transition to metric units, it will, of course,

be necessary in practice to make frequent conversions, but this requirement in no way negates the educational necessity to teach our students to us directly the system which will soon dominate their technical lives.

Included in Appendix A are selected tables of numerical equivalents between US-Customary and SI units. Also included are charts developed by the author for obtaining approximate numerical equivalence between the two systems for selected quantities in mechanics.

IX. Sample calculations. Included in Appendix B are selected elementary problems in mechanics which have been chosen from the areas of statics, mechanics of materials, dynamics, and fluid mechanics to illustrate accepted usage of SI units. The problems are designed to involve most all of the quantities dealt with in mechanics.

X. Selected References

1. Metric Editorial Guide (1974), American National Metric Council, 1625 Massachusetts Ave., N.W., Washington, D.C. 20036

2. Metric Practice Guide (1973) (E 380-72^{e}), American Society for Testing and Materials, 1916 Race St., Philadelphia, PA 19103

3. The International System of Units (SI) (NBS 330 '72 Ed.), Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402

4. ASME Orientation and Guide for Use of Metric Units, American Society of Mechanical Engineers, 345 East 47th St., New York, N.Y. 10017

5. The International System of Units, (NASA SP-7012) 1973, National Aeronautics and Space Administration, Washington, D.C. 20546.

APPENDIX A

CONVERSION FACTORS *

U.S.-British Units to SI Units

To convert from	To	Multiply by
(Acceleration)		
foot/second^2 (ft/s^2)	metre/second^2 (m/s^2)	3.048×10^{-1} •
inch/second^2 (in./s^2)	metre/second^2 (m/s^2)	2.54×10^{-2} •
(Area)		
foot^2 (ft^2)	metre^2 (m^2)	9.2903×10^{-2}
inch^2 (in.^{2})	metre^2 (m^2)	6.4516×10^{-4} •
yard^2 (yd^2)	metre^2 (m^2)	8.3613×10^{-1}
(Density)		
pound mass/inch^3 (lbm/in.^{3})	kilogram/metre^3 (kg/m^3)	2.7680×10^4
pound mass/foot^3 (lbm/ft^3)	kilogram/metre^3 (kg/m^3)	1.6018×10
(Energy, Work)		
British thermal unit (BTU)	joule (J)	1.0551×10^3
foot-pound force (ft · lbf)	joule (J)	1.3558
kilowatt-hour (kw · h)	joule (J)	3.60×10^6 •
(Force)		
kip (1000 lbf)	newton (N)	4.4482×10^3
pound force (lbf)	newton (N)	4.4482
ounce force	newton (N)	2.7801×10^{-1}
(Length)		
foot (ft)	metre (m)	3.048×10^{-1} •
inch (in.)	metre (m)	2.54×10^{-2} •
mile (mi), (U.S. statute)	metre (m)	1.6093×10^3
mile (mi), (international nautical)	metre (m)	1.852×10^3 •
yard (yd)	metre (m)	9.144×10^{-1} •
(Mass)		
pound mass (lbm)	kilogram (kg)	4.5359×10^{-1}
slug (lbf · s^2/ft)	kilogram (kg)	1.4594×10
ton (2000 lbm)	kilogram (kg)	9.0718×10^2
(Power)		
foot-pound/minute (ft · lbf/min)	watt (W)	2.2597×10^{-2}
horsepower (550 ft · lbf/s)	watt (W)	7.4570×10^2
(Pressure, stress)		
atmosphere (std) (14.7 lbf/in.^{2})	newton/metre^2 (N/m^2 or Pa)	1.0133×10^5
pound/foot^2 (lbf/ft^2)	newton/metre^2 (N/m^2 or Pa)	4.7880×10
pound/inch^2 (lbf/in.^{2} or psi)	newton/metre^2 (N/m^2 or Pa)	6.8948×10^3
(Velocity)		
foot/minute (ft/min)	metre/second (m/s)	5.08×10^{-3} •
foot/second (ft/s)	metre/second (m/s)	3.048×10^{-1} •
knot (nautical mi/h)	metre/second (m/s)	5.1444×10^{-1}
mile/hour (mi/h)	metre/second (m/s)	4.4704×10^{-1} •
mile/hour (mi/h)	kilometre/hour (km/h)	1.6093
mile/second (mi/s)	kilometre/second (km/s)	1.6093
(Volume)		
foot^3 (ft^3)	metre^3 (m^3)	2.8317×10^{-2}
inch^3 (in.^{3})	metre^3 (m^3)	1.6387×10^{-5}

*Exact value

Additional Acceptable Units and their SI Equivalents

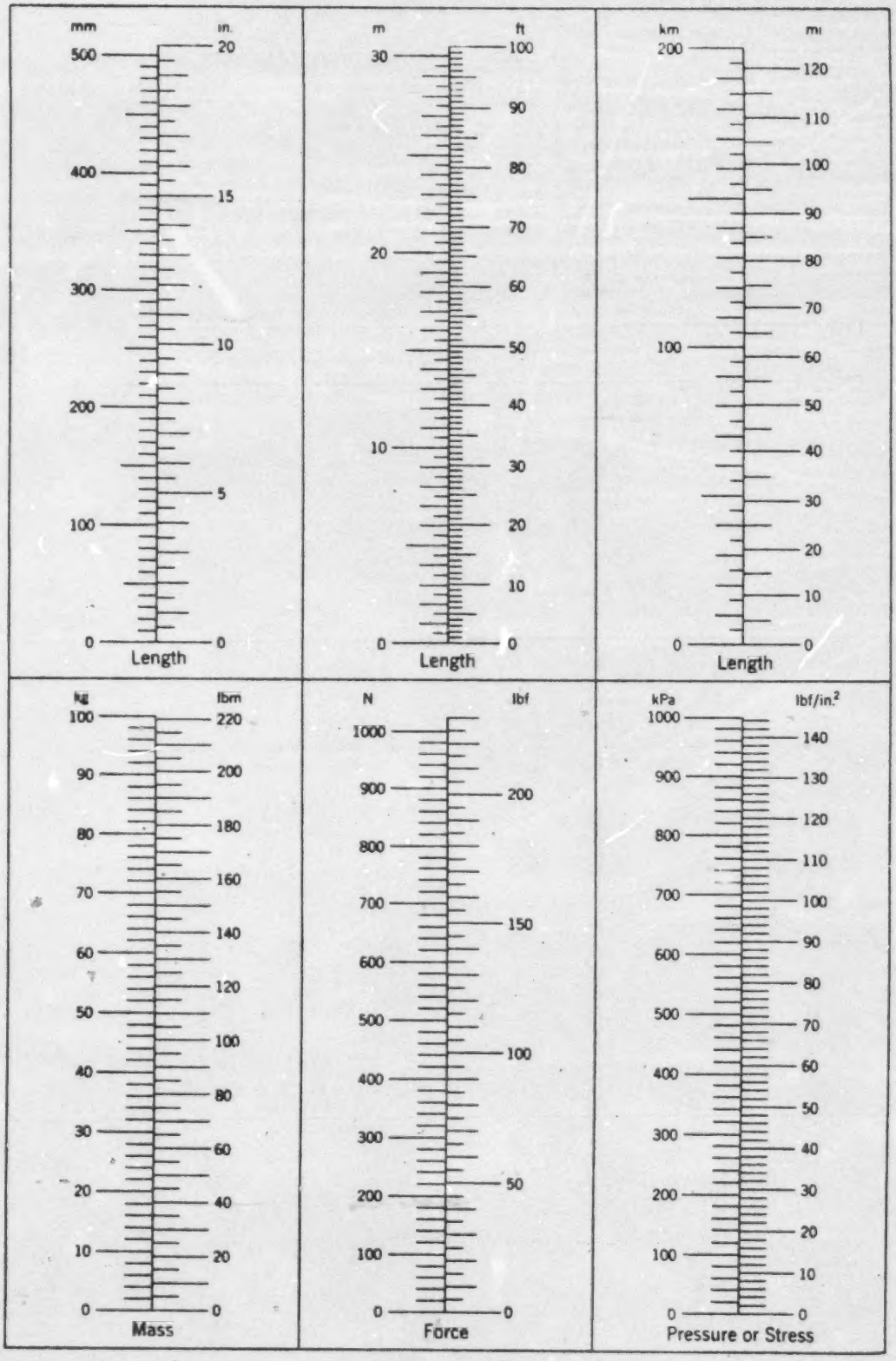
(Frequency)	1 hertz (Hz)	=	1 cycle/second (s^{-1})
(Mass)	1 tonne‡ (metric ton) (t)	=	1000 kilogram (10^3 kg)
(Pressure or stress)	1 bar (b)	=	100 kilopascal (10^2 kPa or 10^5 N/m^2)
	1 pascal (Pa)	=	1 newton/metre^2 (N/m^2)
(Viscosity, absolute)	1 centipoise	=	1 millipascal-second (mPa · s or 10^{-3} N · s/m^2)

‡Pronunciation: rhymes with "John"

* (Taken from the author's books STATICS & DYNAMICS, SI VERSIONS Courtesy of John Wiley & Sons)

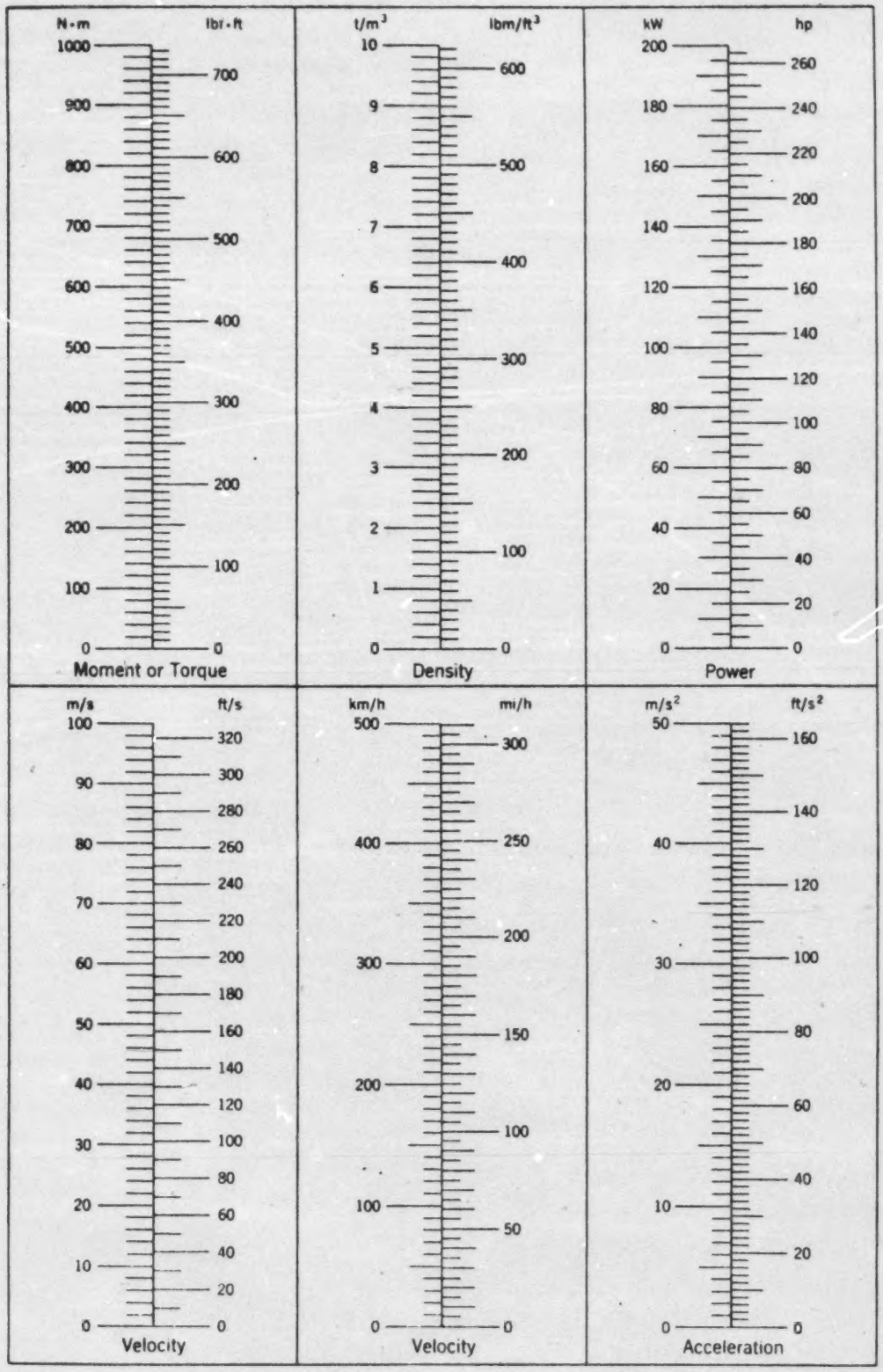
APPENDIX A

Conversion Charts Between SI and U.S.–British Units *



* (Taken from the author's books STATICS & DYNAMICS, SI VERSIONS Courtesy of John Wiley & Sons)

Conversion Charts (con't) *



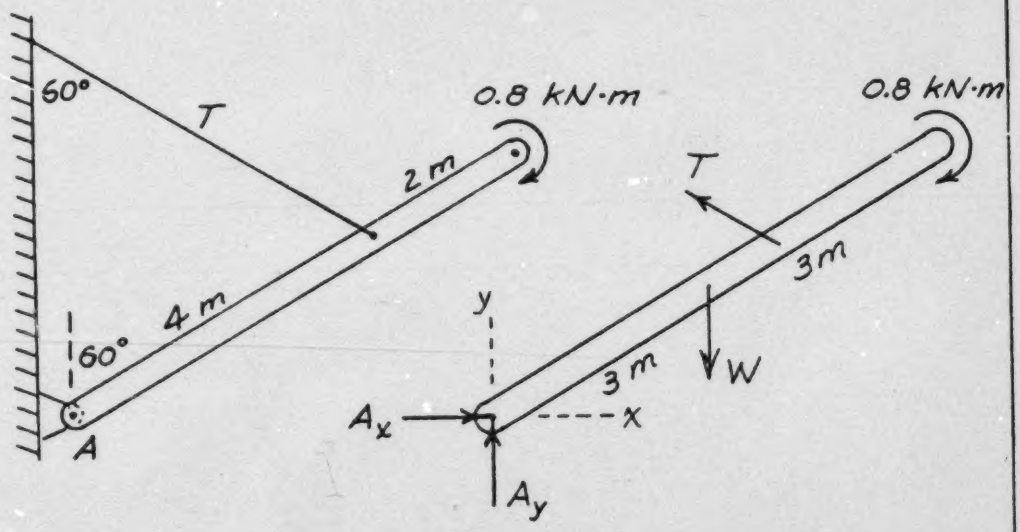
*(Taken from the author's books STATICS & DYNAMICS, SI VERSIONS Courtesy of John Wiley & Sons)

APPENDIX B

SAMPLE CALCULATIONS

STATICS

The uniform 6-m boom has a mass of 60 kg and is loaded and supported in the vertical plane as shown. Determine the tension T in the cable.



$W = mg$

$W = (60\ kg)\left(9.81\ \frac{m}{s^2}\right) = 589\ N$ or $0.589\ kN$

$\Sigma M_A = 0$

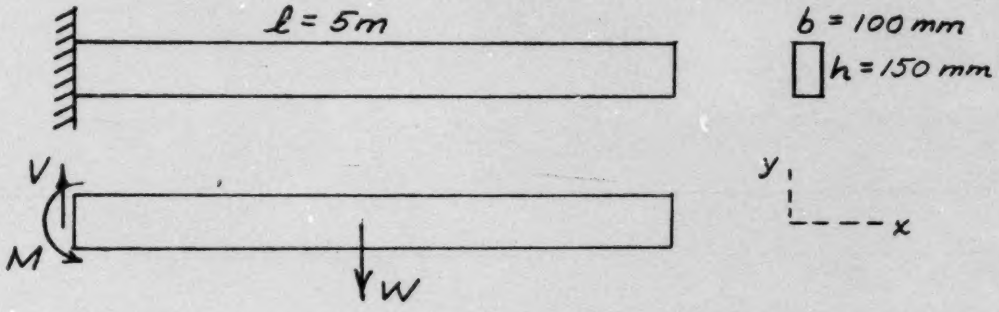
$(0.589\ kN)(3 \sin 60°\ m) + (0.8\ kN \cdot m) - (T \sin 60°)(4\ m) = 0$

$$T = \frac{2.330\ kN \cdot m}{0.866\ (4\ m)} = \underline{0.673\ kN}$$

JLM

STRENGTH OF MATERIALS

Compute the maximum bending stress σ and corresponding strain e for the steel cantilever beam under the action of its own weight. Also find the end deflection y.



$E_{steel} = 200$ GPa or $200(10^9)$ Pa

$\rho_{steel} = 7.83$ t/m^3 or 7830 kg/m^3

$W = mg$

$$W = \left(7830 \frac{kg}{m^3}\right)(5\,m)(0.100\,m)(0.150\,m)\left(9.81 \frac{m}{s^2}\right)$$

$$= 5760\ N$$

$M = W\frac{\ell}{2}$

$$M = (5760\ N)\left(\frac{5}{2}\,m\right) = 14\,400\ N \cdot m$$

$\sigma = \frac{Mc}{I}$

where $c = h/2 = 0.075\,m$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.1(10^6)\ mm^4$$

or $28.1(10^{-6})\ m^4$

$$\sigma = \frac{(14\,400\ N \cdot m)(0.075\,m)}{28.1(10^{-6})\,m^4} = 38.4(10^6)\ Pa$$

or $\underline{\sigma = 38.4\ MPa}$

$y = -\frac{w\ell^4}{8EI}$

$$y = -\frac{W\ell^3}{8EI} = -\frac{(5760\ N)(5\,m)^3}{8\left(200 \times 10^9 \frac{N}{m^2}\right)(28.1 \times 10^{-6})\,m^4}$$

$$= -0.0160\,m \quad \text{or} \quad \underline{-16.0\ mm}$$

$e = \sigma/E$

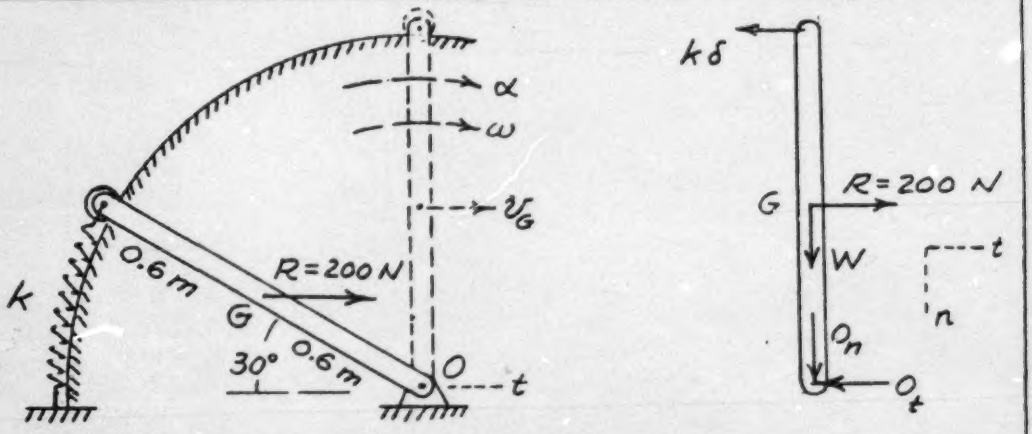
$$e = \frac{38.4\ MPa}{200 \times 10^3\ MPa} = \underline{0.192 \times 10^{-3}}$$

ILM

DYNAMICS

The uniform 10-kg bar is initially at rest in the position shown with the spring of stiffness $k = 40$ N/m unstretched. The bar rotates in the vertical plane about O under the action of the force $R = 200$ N which remains constant in magnitude and direction. As the bar reaches the vertical position determine

(a) ω, v_G
(b) α
(c) t-componet of force at O
(d) power developed by R
(e) linear momentum of bar
(f) angular " " "



$I_O = \frac{1}{3} m\ell^2$

$I_O = \frac{1}{3}(10\text{ kg})(1.2\text{ m})^2 = 4.80\text{ kg}\cdot\text{m}^2$

$W = mg$

$W = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$

$\delta = r\theta$

$\delta = (1.2\text{ m})(\pi/3) = 1.257\text{ m}$

For entire system

$U = \Delta T + \Delta V_g + \Delta V_e$

$U = (200\text{ N})(0.6 \times 0.866\text{ m}) = 103.9\text{ J}$

$\Delta T = \frac{1}{2} I_O \omega^2 = (2.40\text{ kg}\cdot\text{m}^2)\,\omega^2$

$\Delta V_g = Wh = (98.1\text{ N})(0.6\text{ m})(1 - 0.5) = 29.4\text{ J}$

$\Delta V_e = \frac{1}{2} k\delta^2 = \frac{1}{2}\left(40\frac{\text{N}}{\text{m}}\right)(1.257\text{ m})^2 = 31.6\text{ J}$

Thus $103.9 = 2.40\omega^2 + 29.4 + 31.6$

$\omega^2 = 17.88$, $\underline{\omega = 4.23\text{ rad/s}}$ (a)

$v = r\omega$

$v_G = (0.6\text{ m})(4.23\text{ rad/s}) = \underline{2.54\text{ m/s}}$

CON'T

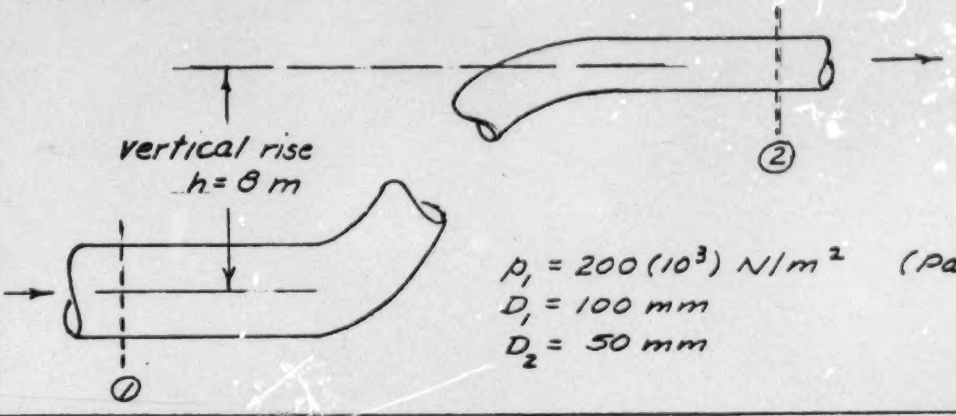
DYNAMICS (CON'T)

$\Sigma M_0 = I_0 \alpha$	$(200\ N)(0.6\ m) - (40\ \frac{N}{m})(1.257\ m)(1.2\ m)$ $= (4.80\ kg \cdot m^2)\,\alpha$
	$\alpha = \frac{59.66\ N \cdot m}{4.80\ kg \cdot m^2} = \underline{12.43\ rad/s^2}$ (b)
$\bar{a}_t = \bar{r}\alpha$	$\bar{a}_t = (0.6\ m)(12.43\ rad/s^2) = 7.46\ m/s^2$
$\Sigma F_t = m\bar{a}_t$	$(200\ N) - (40\ \frac{N}{m})(1.257\ m) - O_t = (10\ kg)(7.46\ \frac{m}{s^2})$
	$O_t = 200 - 60.3 - 74.6 = \underline{75.1\ N}$ (c)
Power $P = RV$	$P = (200\ N)(2.54\ m/s) = 508\ J/s$ $= \underline{508\ W}$ (d)
Linear momentum $G = mv_G$	$G = (10\ kg)(2.54\ m/s) = \underline{25.4\ kg \cdot m/s}$ (or $N \cdot s$) (e)
Angular momentum $H_0 = I_0 \omega$	$H_0 = (4.80\ kg \cdot m^2)(4.23\ rad/s)$ $= \underline{20.3\ kg \cdot m^2/s}$ (or $N \cdot m \cdot s$) (f)

JLM

FLUID MECHANICS

Fresh water flows through the closed pipe at the rate of $1.5\ m^3/min$. If the pressure at section ① is 200 kPa (gage) and the inside pipe diameter is 100 mm, determine the gage pressure at section ② where the inside pipe diameter is 50 mm.



$p_1 = 200(10^3)\ N/m^2$ (Pa)
$D_1 = 100\ mm$
$D_2 = 50\ mm$

$\rho = 1000\ kg/m^3$

$\rho g = (1000\ kg/m^3)(9.81\ m/s^2)$
$= 9810\ kg/(m^2 \cdot s^2)$ (or N/m^3)

Continuity
$Q = A_1 v_1 = A_2 v_2$

$A_1 = \frac{\pi}{4}(0.100)^2 = 78.54(10^{-4})\ m^2$

$A_2 = \frac{\pi}{4}(0.050)^2 = 19.64(10^{-4})\ m^2$

$v_1 = \frac{1.5\ m^3/min}{60\ s/min}\ \frac{1}{78.54(10^{-4})\ m^2} = 3.183\ \frac{m}{s}$

$v_2 = v_1 \frac{A_1}{A_2} = 3.183\left(\frac{4}{1}\right) = 12.73\ m/s$

Conservation of energy
$p + \rho g h + \frac{1}{2}\rho v^2 = const.$

$E_1 = E_2$

$200(10^3)\ \frac{N}{m^2} + 0 + \frac{1}{2}\left(1000\ \frac{kg}{m^3}\right)\left(3.183\ \frac{m}{s}\right)^2$

$= p_2 + \left(9810\ \frac{kg}{m^2 \cdot s^2}\right)(8\ m) + \frac{1}{2}\left(1000\ \frac{kg}{m^3}\right)\left(12.73\ \frac{m}{s}\right)^2$

$200(10^3) + 0 + 5.07(10^3)$
$= p_2 + 78.48(10^3) + 81.03(10^3)$

$p_2 = \underline{45.53(10^3)\ Pa}$ or $\underline{45.53\ kPa}$

JLM